

PROBABILITY

The probability of an outcome of a random variable is the proportion of times the outcome would occur in a very long series of experiments.

Random Variable: **X** (in-the-shade-temperature at noontime in degrees Fahrenheit)

Probability: **P(X=x)** Example: **P(X=72°F)**

These are numbers Example: **P(X=72 ± 2°F) = 25%**

How do we estimate these probability values?

- Unstructured Random Phenomena**

Examples

Temperature at noontime

Effects of a drug (what is the prob. of a headache from use of the drug *Q22*?)

Post-operative complications (prob. patient will be in various states of wellness)

Stock Market (prob. that market will be up by 200 points by January)

Sales (prob. that the company will gross \$20,000,000 in December)

Can *estimate* probabilities by getting a lot of data -- call it a **Method I** study
e.g. conduct an experiment on a large sample of subjects on the effects of *Q22*

Side effects:	Head-ache	Loss of appetite	Fever	Allergic reaction	No Side Effects	Total
# of subjects	205	167	86	63	479	1000
Prob() (estimated)	0.205	0.167	0.086	0.063	0.479	1.0

Patients treated with *Q22* have a 20% chance of getting *Headache* (Oversimplified example assumes mutually exclusive events)

...or "*guesstimate*" probabilities based on experience -- call it **Method II**
(Also known as *personal probabilities/subjective probabilities*)

Oil drilling

example

sum terms to get mean $\mu \rightarrow$

sum to get Variance $s^2 \rightarrow$

class	A	B	C	D	Sum
Yield	100,000	300,000	500,000	Dry Well	
Pr	.3	.2	.1	.4	
X P(X)	30,000	60,000	50,000		
$(X-\mu)^2 P(X)$					

We feel that sales will exceed \$20 million with probability of 90%
The odds are 3:1 that the Yankees will beat the Orioles

Structured Random Phenomena

Examples

Toss of a coin	P(X = heads) = 1/2
Roll of a die	P(X = 3) = 1/6

Probability values are derived logically -- **Method III** (structured prob.)

PROBABILITY MODELS

"Outcome" -- *Examples:*

coin comes up heads, die X=3, temp= 72°F

"Event" is a set of one or more outcomes -- *Examples:*

coin comes up heads, die X=3 or 4, temp ≥ 72°F

Probability Rules:

- $0 \leq P(\text{Event}) \leq 1$

- $\sum P(\text{outcome}) = 1$ (summing over all outcomes)

- $P(\text{Event doesn't occur}) = 1 - P(\text{Event does occur})$

If 2 Events have no common outcomes:

$$P(\text{Event I or Event II}) = P(\text{Event I}) + P(\text{Event II})$$

If 2 Events have common outcomes: P(Event I or Event II)

$$= P(\text{Event I}) + P(\text{Event II}) - P(\text{Event I and Event II})$$

Exp. Values: $\sum \text{Outcome} \times P(\text{Outcome})$ (summing over all outcomes)

Mean of X: $\sum X P(X) = \mu$ (summing over all X)

Variance: $\sum (X-\mu)^2 P(X) = s^2$ (summing over all X)